Shallow Water Acoustic Propagation at Arraial do Cabo, Brazil

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Abstract

The Brazilian coastal region is a huge platform where the propagation is dominantly in a waveguide delimited by the surface and the bottom. The region near Arraial do Cabo is a place where there is a special sound speed profiles (SSP) regimes related to shallow waters. For some particular environmental and signal parameters, shallow water means a region where the acoustic waveguide propagation occurs for low as well as intermediary frequencies. This work aims to describe some characteristics of waveguide propagation with typical sound speed profile of this place. The modal and ray tracing theories were described with experimental results. Discussion about how the fluctuations of environmental parameters would affect the waveguide in shallow waters were introduced.

Keywords: ***underwater acoustics, shallow water acoustics, waveguide propagation, ray tracing***

Introduction

One of the characteristics of shallow waters are sound speed profile refracting downwards or nearly constant, meaning that long range propagation takes place exclusively via bottom-interacting paths, these shallow water environments are found on the continental shelf for waters depths down to 200 meters. Sound is used widely in undersea applications because the ocean is essentially transparent to acoustic waves, whereas it is opaque to all types of electromagnetic waves. Applications of underwater sound include (Buckingham, 1992) active and passive detection of ships, submarines, seismic profiling, echo-sounding, high-resolution imaging, communications and acoustic tomography. In Low frequency regime, long range sound transmissions across oceans are possible. This ocean feature have been proposed as a mean of monitoring long-term global change (Buckingham, 1992), as well as the sound generated naturally at the surface is being considered as the basis of remote sensing methods for estimating weather conditions over ocean large scale. In the next sections we try to explain the widely used theories to sound propagation in shallow water. To this end, we outline this work in the following form, first we introduce the normal mode and ray tracing theories and in the second section we show our experimental results in a region of Brazil where there is a special regime of sound speed profile, the region of Arraial do Cabo. Finally in the last section we discuss and summarize our main results.

Theory

The intensity and phase of the sound field generated by a acoustic source in the ocean can be deduced by solving the Helmholtz wave equation. The complexity of the acoustic environment like sound speed profile, the roughness of the sea surface, the stratification of the sea floor and the internal waves introduce acoustic fluctuations during the sound propagation that decrease the signal finesse (Buckingham, 1992). Nowadays several types of solutions for the sound pressure field are available, in this work we deal with two of them namely: normal mode and ray tracing techniques. The most important acoustical parameter of the ocean is the speed of sound and it depends on the following oceanographic parameters, the temperature, salinity and pressure (Clay, 1977). The average speed at sea is approximately 1500 m/s and have spatial and temporal variations. Besides, based on experimental data of the above physical parameters, it is possible to obtain some empirical formulas for sound speed in the ocean (Clay, 1977), namely:

$$\begin{aligned} c = 1449.2 + 4.6T - 0.055T^2 + \\ 0.00029T^3 + (1.34 - 0.010T)(S-35) + 0.016z \end{aligned} \tag{1}$$

where c=speed(m/s),T=temperature(^{o}C), S=salinity(parts per thousand), z=depth(meters).

In the ideal waveguide normal mode propagation framework, for a given frequency f, the harmonic pressure fields $\Phi(\mathbf{r})exp(-i\omega t)$ $(\omega = 2\pi f)$ propagating with speed $c(\mathbf{z})$ are solution of the Helmholtz's wave equation:

$$\left(\bigtriangledown^2 + \frac{\omega^2}{c(\mathbf{z})^2}\right)\Phi(\mathbf{r}) = 0 \tag{2}$$

Besides, adopting the circular cylindrical coordinate system (where the depth increases in the positive z direction) and assuming the ocean a system with azimuthal symmetry, then the regular solutions of (2) can be written as:

$$\Phi(\mathbf{r}) \equiv \int_0^\infty dk\, A(k)kJ_0(kr)\Psi(z,k). \tag{3}$$

(Sommerfeld-Weyl Picture)

Where k is the radial wave number, J_0 is the zero–th order Bessel function. In addition, *"the vertical wave function"* Ψ satisfies the following wave equation:

$$\frac{d^2\Psi}{dz^2} + \left(\frac{\omega^2}{c(z)^2} - k^2\right)\Psi = 0. \tag{4}$$

Applying Cauchy's residue theorem, the Sommerfeld-Weyl integral (3) can be rewritten as:

$$\Phi(\mathbf{r}) \equiv \int_0^\infty dk\, A(k) k J_0(kr) \Psi(z,k) \quad \underbrace{2\pi i \sum_n \text{Residue Series}}_{\text{Normal Modes: Discrete Spectrum}} + \underbrace{\frac{1}{2} \int_{\Gamma\in\mathbb{C}} dk\, A(k)\; k\; H_0^{(1)}(kr)\Psi(z,k)}_{\text{Continuum Spectrum}} \tag{5}$$

Asymptotic *Ray Tracing* expansion theory:

$$\Phi(\mathbf{r}) = e^{i\omega\,\tau(\mathbf{r})} \sum_{j=0}^{\infty} \frac{A_j(\mathbf{r})}{(i\omega)^j} \tag{6}$$

$$\begin{aligned} |\nabla\tau|^2 &= c^{-2}(\mathbf{r}) + O(\omega^2)\,\text{(Eikonal)} \\ 2\nabla\tau\cdot\nabla A_0 + (\nabla^2\tau)A_0 &= 0 + O(\omega)\ \text{(Amplitud)} \end{aligned}$$

Rays are the wave front orthogonal curves family, $\omega\,\tau(\mathbf{r}) \equiv$ Constant such that,

$$\frac{d\,\mathbf{r}}{d\,s} = c\nabla\tau. \tag{7}$$

Finally, it is necessary to calculated in dB the Transmission Loss (TL), given by:

$$TL(\mathbf{r}) = -20\log\left|\frac{\Phi(\mathbf{r})}{\Phi_0}\right| \tag{8}$$

For instance, the normal mode solution in an ideal waveguide where the surface is perfectly free and the bottom perfectly rigid, the boundaries are perfect sound reflectors at all angles of incidence. The sound speed c_w and depth are constant. As a consequence of this particular situation, the values of the pressure reflection coefficient at the surface and at the sea bottom are ±1 respectively (Jessen, 1994). In this framework, the residue series in (5) gives the following sound pressure field as the following sum over all propagating modes (Jessen, 1994).

$$p = a_0 \exp\left[i\left(\omega t + \pi/4\right) \sum_m \frac{\rho_0 Z_m(z_0) Z_m(z) \exp(-i\kappa_m r)}{\upsilon_m (2\pi\kappa_m r)^{1/2}}\right. \tag{9}$$

In the present situation, where we assume harmonic fields, the above equation(9) is a explicit function of the angular frequency $\omega = 2\pi f$ and the propagation time t. In addition, the normal mode expression(9) of the sound pressure is suitable to calculating sound field as a function of depth z and range r, where the horizontal κ and vertical γ wave numbers satisfy the following equation:

$$\kappa^2 = \kappa_m^2 + \gamma_m^2 = \frac{\omega^2}{c^2}. \tag{10}$$

Besides, in (9) a_o is a constant related to the source power, ρ_w is the water density, z_0 is the source depth, Z_m is the vertical eigenfunction, while υ_m is proportional to the mean energy flux passing through a vertical section. For sound with a wavelength λ propagating in an ideal wave guide of depth d, it is possible estimate R_{max} the maximum distance separation between the sound source and receiver (Katsnelson, 2002), namely:

$$R_{max} = 2d^2/\lambda \tag{11}$$

In general, due to wave guide geometry and bottom geological features some frequencies are dropped. These allowed frequencies are limited by a cut off frequency f_o, where a roughly estimate (Brekhovskikh, 2002) to f_o is given by:

$$f_o = \frac{c_w}{4d\sin\theta_c}, \tag{12}$$

being the incident critical angle $\theta_c = arccos(c_w/c_b)$ and cw, cb the mean values of sound speed along the water column and the sea bottom respectively. Notice that the frequency f_o(12) is strongly depend on water depth. In another words, the f_o numerical value is a general feature of ducted propagation and it occurs as a result of the evanescent wave propagation into the sea bottom. For instance, the efficiency of the duct to confine sound decreases as f_o(12) increases. On the other hand, for a given frequency f the number m of propagating modes in (9) is finite and satisfies the following inequality (Jessen, 1994):

$$m \leq \frac{f}{c_w}(2d\cos\theta_c) + 1/2 \tag{13}$$

Equation(13) shows that as the frequency f increases the number of propagating modes increases (Jessen, 1994).

Keeping these ideas in mind, in the next section we analyze theoretically our experimental results.

Results

The ray solution of wave equation(6) is a high frequency approximation (Buckingham, 1992) and this approach has a special significance for deep waters problems where only few rays are significant. The ray tracing methods are more suitable when the wavelength is very much less than any length scale related to the problem. These lengths include bottom and surface roughness, the dimensions of the campaign region as well as the distance over measurable changes in the sound speed profile. There is also some advantages in ray tracing, like velocity of computing rays can be traced through range-dependent sound speed profile and over complicated batimetry. The difficulties are keeping the track of the phase at bottom reflections, and computing shear waves contributions in an elastic bottom, but the principal disadvantage is that the wave effects such as diffraction and caustics cannot be handled satisfactorily. So, in order to obtain some physical insight about the main features of the sound propagation, it is suitable analyze our results on both frameworks, the ray tracing as well as normal mode theory.

The sound speed profile sketched in Figure 1 is typical from Arraial do Cabo,RJ, Brazil. Here, it represents one month averaged sound speed profile. In addition in this same geographical region, assuming a constant depth of d= 80m, the structure of the water layer and the first sub bottom sediment are estimated by the following values of the constitutive parameters ρ_w=1024 kg/m^3, c_b=1750 m/s, $\rho_b = 1800kg/m^3$ respectively. However, in our numeric codes, as a first approximation we do not take into account any bottom absorption effects. Assuming these above acoustic propagation parameters values, the figure 2 shows for a source located at the depth of 10m, the related ray tracing acoustic energy propagation theory.

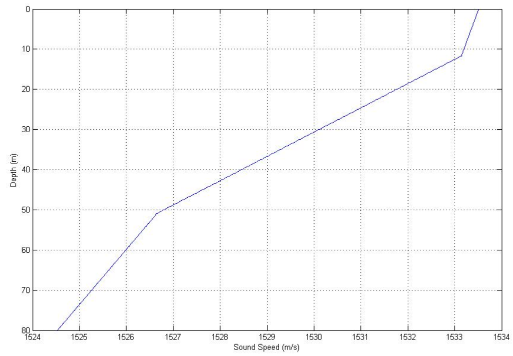

Figure 1: Arraial do Cabo sound speed profile

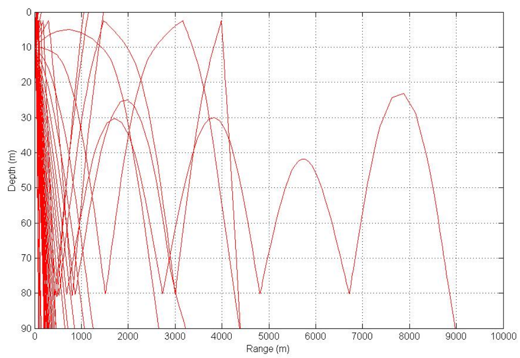

Figure 2: Ray tracing propagation in Arraial do Cabo shallow waters.

Related to same region in Arraial do Cabo, we obtain the following numerical estimated values to equations(12 and 13) namely, fo=27Hz, $m \leq 91$ and an incident critical angle of $\theta_c = 29^o$. Assuming a point source at the depth d=10m radiating with frequency f=1kHz ($\lambda = 1.53m$), figure(3) shows a color map (in dB) related to the behavior of the transmission loss (8), as function of the range and the depth. It is interesting to note that both figures(2 and 3) suggest a long range guided sound propagation into the water layer located at the depths $10m \leq z \leq 50m$. On the other hand, figure 1 shows that this layer is related to strong negative gradient variations on sound speed profile at the depths $z \approx 10m$ and $z \approx 50m$ respectively.

Summary and Conclusions

In this work based on the normal mode and ray tracing theories, we have analyzed some acoustic propagation features in Arraial do Cabo shallow waters. Ray as well as normal modes theories are widely used for the description of waveguide sound propagation in the ocean. But if the sound wavelength can be compared to ocean depth, the number of significant modes become very large and it is necessary to apply a modal theory to accurately analyze important wave propagation features

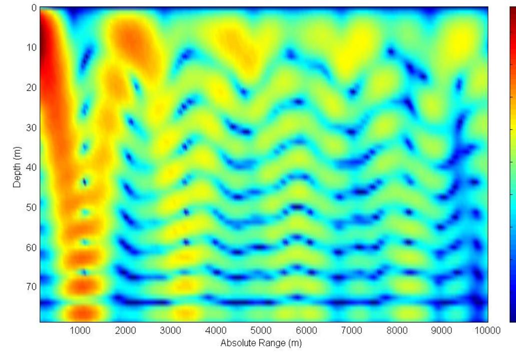

Figure 3: Sound modal propagation in Arraial do Cabo shallow waters

such as caustics (compare figure 2 and 3), interference and diffraction (see bright and shadow zones in figure 3). In the present situation, the number of excited modes is about $m \approx 91$ (see eq 13) and both theories should be applied. Besides, it is possible to see in figures (2 and 3) that the negative gradient sound speed profile (see figure 1) introduces a strong downward refraction in acoustic energy propagation. This Arraial do Cabo environmental feature permits a ducted sound propagation, but any change in the parameters like water salinity and temperature (see eq 1) would affect the waveguide behavior, although the structure of the bottom and sub bottom stay the same. On the other hand, if the acoustic experiment take place in a new near region, but with different structure of the bottom and sub bottom, this could affect the waveguide stability either, since the critical angle θ_c is very sensitive to bottom features.

Summarizing, the results of the present paper related to Arraial do Cabo experimental sound profile data, suggest the possibility of long range acoustic propagation. However, the maintenance of this shallow water acoustic waveguide is strongly dependent on the environmental conditions.

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